1.1

8. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP ; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP ; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP . Determine the truth value of each of these propositions.

a) Smartphone B has the most RAM of these three smartphones.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

d) If Smartphone B has more RAM and more ROM than Smartphone C , then it also has a higher resolution camera.

e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

9. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.

b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.

d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

1.2

Exercises 28-35 relate to inhabitants of an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies (called normals by Smullyan [Sm78]) who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For each of these situations, if possible, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

28. A says " C is the knave," B says " A is the knight," and C says "I am the spy."
29. A says "I am the knight," B says "I am the knave," and C says " B is the knight."
30. A says "I am the knave," B says "I am the knave," and C says "I am the knave."
31. A says "I am the knight," B says " A is telling the truth," and C says "I am the spy."
32. A says "I am the knight," B says " A is not the knave," and C says " B is not the knave."
33. A says "I am the knight," B says "I am the knight," and C says "I am the knight."
34. A says "I am not the spy," B says "I am not the spy," and C says " A is the spy."
35. A says "I am not the spy," B says "I am not the spy," and C says "I am not the spy."

Exercises 36-42 are puzzles that can be solved by translating statements into logical expressions and reasoning from these expressions using truth tables.

36. The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if

a) one of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?

b) innocent men do not lie?

1.3

38. Find the dual of each of these compound propositions.

a) $p \lor \neg q$

b) $p \land (q \lor (r \land T))$

c) $(p \land \neg q) \lor (q \land F)$

39. Find the dual of each of these compound propositions.

a) $p \land \neg q \land \neg r$

b) $(p \land q \land r) \lor s$

c) $(p \lor F) \land (q \lor T)$

40. When does $s^* = s$, where s is a compound proposition?

41. Show that $(s^*)^* = s$ when s is a compound proposition.

42. Show that the logical equivalences in Table 6, except for the double negation law, come in pairs, where each pair contains compound propositions that are duals of each other.

43. Why are the duals of two equivalent compound propositions also equivalent, where these compound propositions contain only the operators $\land$,$\lor$, and $\neg$?

47. Show that $\neg$,$\land$, and $\lor$ form a functionally complete collection of logical operators. [Hint: Use the fact that every compound proposition is logically equivalent to one in disjunctive normal form, as shown in Exercise 46.]

48. Show that $\neg$ and $\land$ form a functionally complete collection of logical operators. [Hint: First use a De Morgan law to show that $p \lor q$ is logically equivalent to $\neg(\neg p \land \neg q)$.]

49. Show that $\neg$ and $\lor$ form a functionally complete collection of logical operators.

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \land \mathbf{T} \equiv p$ $p \lor \mathbf{F} \equiv p$	Identity laws
$p \lor \mathbf{T} \equiv \mathbf{T}$ $p \land \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \lor p \equiv p$ $p \land p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	Commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	Associative laws
$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$ $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$	Distributive laws
$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$	De Morgan's laws
$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption laws
$p \lor \neg p \equiv \mathbf{T}$ $p \land \neg p \equiv \mathbf{F}$	Negation laws

1.4

48. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
a) $(\forall x P(x)) \vee A \equiv \forall x(P(x) \vee A)$
b) $(\exists x P(x)) \vee A \equiv \exists x(P(x) \vee A)$
49. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
a) $(\forall x P(x)) \wedge A \equiv \forall x(P(x) \wedge A)$
b) $(\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$
50. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
a) $\forall x(A \rightarrow P(x)) \equiv A \rightarrow \forall x P(x)$
b) $\exists x(A \rightarrow P(x)) \equiv A \rightarrow \exists x P(x)$
51. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
a) $\forall x(P(x) \rightarrow A) \equiv \exists x P(x) \rightarrow A$
b) $\exists x(P(x) \rightarrow A) \equiv \forall x P(x) \rightarrow A$

Exercises 61-64 are based on questions found in the book Symbolic Logic by Lewis Carroll.
61. Let $P(x), Q(x)$, and $R(x)$ be the statements " x is a professor," " x is ignorant," and " x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x)$, and $R(x)$, where the domain consists of all people.
a) No professors are ignorant.
b) All ignorant people are vain.
c) No professors are vain.
d) Does (c) follow from (a) and (b)?
62. Let $P(x), Q(x)$, and $R(x)$ be the statements " x is a clear explanation," " x is satisfactory," and " x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and $P(x), Q(x), and R(x)$.
a) All clear explanations are satisfactory.
b) Some excuses are unsatisfactory.
c) Some excuses are not clear explanations.
*d) Does (c) follow from (a) and (b)?
63. Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements " x is a baby," " x is logical," " x is able to manage a crocodile," and " x is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x)$, $R(x)$, and $S(x)$.
a) Babies are illogical.
b) Nobody is despised who can manage a crocodile.

c) Illogical persons are despised.

d) Babies cannot manage crocodiles.

*e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

64. Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements " x is a duck," " x is one of my poultry," " x is an officer," and " x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

a) No ducks are willing to waltz.

b) No officers ever decline to waltz.

c) All my poultry are ducks.

d) My poultry are not officers.

*e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

1.5

29. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1,2 , or 3 and y is 1,2 , or 3 . Write out these propositions using disjunctions and conjunctions.

a) $\forall x \forall y P(x, y)$

b) $\exists x \exists y P(x, y)$

c) $\exists x \forall y P(x, y)$

d) $\forall y \exists x P(x, y)$

30. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a) $\neg \exists y \exists x P(x, y)$

b) $\neg \forall x \exists y P(x, y)$

c) $\neg \exists y \big(Q(y) \wedge \forall x \neg R(x, y)\big)$

d) $\neg \exists y \big(\exists x R(x, y) \vee \forall x S(x, y)\big)$

e) $\neg \exists y \big(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z)\big)$

31. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\forall x \exists y \forall z T(x, y, z)$

b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

c) $\forall x \exists y \big(P(x, y) \wedge \exists z R(x, y, z)\big)$

d) $\forall x \exists y \big(P(x, y) \rightarrow Q(x, y)\big)$

32. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\exists z \forall y \forall x T(x, y, z)$

b) $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

c) $\exists x \exists y \big(Q(x, y) \leftrightarrow Q(y, x)\big)$

d) $\forall y \exists x \exists z \big(T(x, y, z) \vee Q(x, y)\big)$

48. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x \forall y \big(P(x) \vee Q(y)\big)$, where all quantifiers have the same nonempty domain, are logically equivalent. (The new variable y is used to combine the quantifications correctly.)

49. a) Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x \exists y \big(P(x) \wedge Q(y)\big)$, where all quantifiers have the same nonempty domain.

b) Show that $\forall x P(x) \vee \exists x Q(x)$ is equivalent to $\forall x \exists y \big(P(x) \vee Q(y)\big)$, where all quantifiers have the same nonempty domain.

1.6

4. What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

9. For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."

b) "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

c) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."

d) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

e) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

f) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

23. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x\big(P(x) \wedge Q(x)\big)$ is true.

1. $\exists x P(x) \vee \exists x Q(x)$ Premise
2. $\exists x P(x)$ Simplification from (1)
3. $P(c)$ Existential instantiation from (2)
4. $\exists x Q(x)$ Simplification from (1)
5. $Q(c)$ Existential instantiation from (4)
6. $P(c) \wedge Q(c)$ Conjunction from (3) and (5)
7. $\exists x\big(P(x) \wedge Q(x)\big)$ Existential generalization

26. Justify the rule of universal transitivity, which states that if $\forall x\big(P(x) \rightarrow Q(x)\big)$ and $\forall x\big(Q(x) \rightarrow R(x)\big)$ are true, then $\forall x\big(P(x) \rightarrow R(x)\big)$ is true, where the domains of all quantifiers are the same.

34. The Logic Problem, taken from $WFF^{'}\ N$ PROOF, The Game of Logic, has these two assumptions:

1. "Logic is difficult or not many students like logic."
2. "If mathematics is easy, then logic is not difficult."

By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

a) That mathematics is not easy, if many students like logic.
b) That not many students like logic, if mathematics is not easy.
c) That mathematics is not easy or logic is difficult.
d) That logic is not difficult or mathematics is not easy.
e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

1.7

19. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
a) a proof by contraposition.
b) a proof by contradiction.

27. Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$. [Hint: Assume that $r = a / b$ is a root, where a and b are integers and a / b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

36. Is this reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1} = x$ correct? (1) $\sqrt{2x^2 - 1} = x$ is given; (2) $2x^2 - 1 = x^2$, obtained by squaring both sides of (1); (3) $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2); (4) $(x - 1)(x + 1) = 0$, obtained by factoring the left-hand side of $x^2 - 1$; (5) $x = 1$ or $x = -1$, which follows because $a\,b = 0$ implies that $a = 0$ or $b = 0$.

1.8

28. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [Hint: Work backward, assuming that you did end up with nine zeros.]

39. Let $S = x_1y_1 + x_2y_2 + \cdots + x_ny_n$, where $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are orderings of two different sequences of positive real numbers, each containing n elements.
a) Show that S takes its maximum value over all orderings of the two sequences when both sequences are sorted (so that the elements in each sequence are in nondecreasing order).
b) Show that S takes its minimum value over all orderings of the two sequences when one sequence is sorted into nondecreasing order and the other is sorted into nonincreasing order.

48. Prove that when a white square and a black square are removed from an 8×8 checkerboard (colored as in the text) you can tile the remaining squares of the checkerboard using dominoes. [Hint: Show that when one black and one white square are removed, each part of the partition of the remaining cells formed by inserting the barriers shown in the figure can be covered by dominoes.]

12.1

24. Simplify these expressions.

a) $x \oplus 0$

b) $x \oplus 1$

c) $x \oplus x$

d) $x \oplus \bar{x}$

25. Show that these identities hold.

a) $x \oplus y = (x + y)\overline{(xy)}$

b) $x \oplus y = (x\bar{y}) + (\bar{x}y)$

26. Show that $x \oplus y = y \oplus x$.

27. Prove or disprove these equalities.

a) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

b) $x + (y \oplus z) = (x + y) \oplus (x + z)$

c) $x \oplus (y + z) = (x \oplus y) + (x \oplus z)$

28. Find the duals of these Boolean expressions.

a) $x + y$

b) $\bar{x}\bar{y}$

c) $xyz + \bar{x}\bar{y}\bar{z}$

d) $x\bar{z} + x \cdot 0 + \bar{x} \cdot 1$

12.2

5. Find the sum-of-products expansion of the Boolean function $F(w, x, y, z)$ that has the value 1 if and only if an odd number of w, x, y, and z have the value 1 .

14. Show that

a) $\bar{x} = x \mid x$.

b) $xy = (x \mid y) \mid (x \mid y)$.

c) $x + y = (x \mid x) \mid (y \mid y)$.

15. Show that

a) $\bar{x} = x \downarrow x$.

b) $xy = (x \downarrow x) \downarrow (y \downarrow y)$.

c) $x + y = (x \downarrow y) \downarrow (x \downarrow y)$.

12.3

6. Construct circuits from inverters, AND gates, and OR gates to produce these outputs.

a) $\bar{x} + y$

b) $\overline{(x + y)}x$

c) $xyz + \bar{x}\bar{y}\bar{z}$

d) $\overline{(\bar{x} + z)(y + \bar{z})}$

10. Construct a circuit for a half subtractor using AND gates, OR gates, and inverters. A half subtractor has two bits as input and produces as output a difference bit and a borrow.

11. Construct a circuit for a full subtractor using AND gates, OR gates, and inverters. A full subtractor has two bits and a borrow as input, and produces as output a difference bit and a borrow.

15. Use NAND gates to construct circuits with these outputs.

a) $\bar{x}$

b) $x + y$

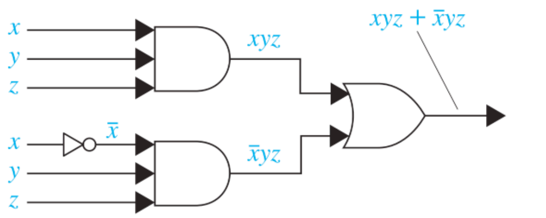
c) $x\, y$

d) $x \oplus y$

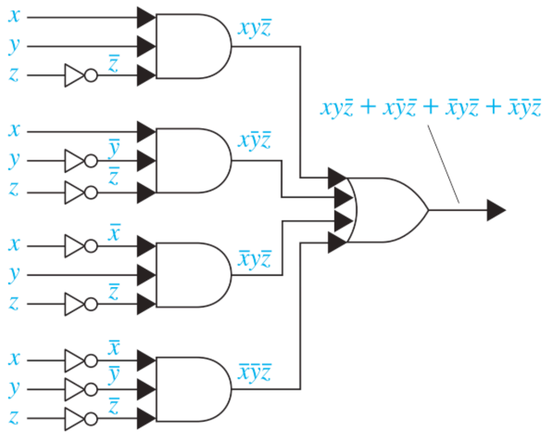
12.4

6. Use K-maps to find simpler circuits with the same output as each of the circuits shown.

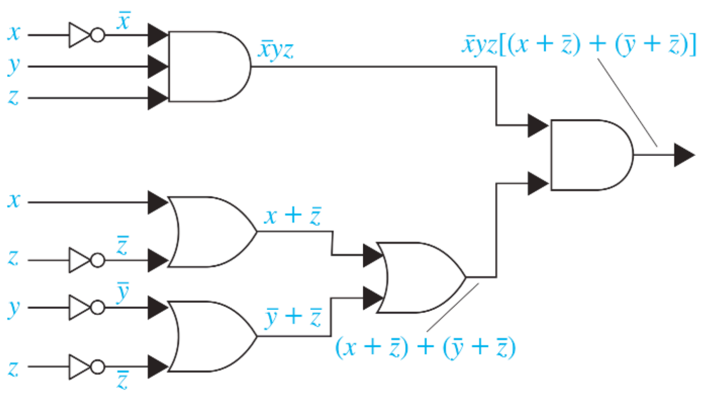
a)



b)



c)



12. Use a K-map to find a minimal expansion as a Boolean sum of Boolean products of each of these functions in the variables x, y, and z.

a) $\bar{x}yz + \bar{x}\bar{y}z$

b) $xyz + xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z}$

c) $xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z$

d) $xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

In Exercises 30-31 find a minimal sum-of-products expansion, given the K-map shown with don't care conditions indicated with ds.

30.

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	d	1	d	1
$w\bar{x}$		d	d	
$\bar{w}\bar{x}$		d	1	
$\bar{w}x$		1	d	

31.

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	1			1
$w\bar{x}$		d	1	
$\bar{w}\bar{x}$		1	d	
$\bar{w}x$	d			d

Chapter 1-Summary

1. What is the truth value of $(p \vee q) \rightarrow (p \wedge q)$ when both p and q are false?
2. Write the converse and contrapositive of the statement "If it is sunny, then I will go swimming."
3. Show that $\neg(p \vee \neg q)$ and $q \wedge \neg p$ are equivalent
(a) using a truth table.
(b) using logical equivalences.
4. Suppose that $Q(x)$ is the statement " $x+1=2x$." What are the truth values of $\forall x Q(x)$ and $\exists x Q(x)$?
5. Prove each of the following statements.
(a) The sum of two even integers is always even.
(b) The sum of an even integer and an odd integer is always odd.
6. Prove that there are no solutions in positive integers to the equation $x^4+y^4=100$.
7. Prove or disprove that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are equivalent.
8. Let $P(m,n)$ be " n is greater than or equal to m " where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of $\exists n \forall m P(m,n)$ and $\forall m \exists n P(m,n)$?
9. Prove that all the solutions to the equation $x^2=x+1$ are irrational.
10. (a) Prove or disprove that a 6×6 checkerboard with four squares removed can be covered with straight triominoes.
(b) Prove or disprove that an 8×8 checkerboard with four squares removed can be covered with straight triominoes.
11. A stamp collector wants to include in her collection exactly one stamp from each country of Africa. If $I(s)$ means that she has stamp s in her collection, $F(s,c)$ means that stamp s was issued by country c, the domain for s is all stamps, and the domain for c is all countries of Africa, express the statement that her collection satisfies her requirement. Do not use the $\exists!$ symbol.

Chapter 12-Summary

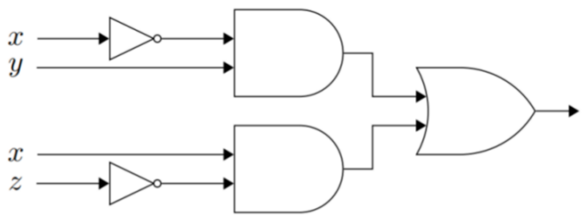
1. What is the value of the Boolean function $f(x,y,z) = (\bar{x}+\bar{y})z + xyz$ when $x = 1, y = 0\ and\ z = 1$?

2. Prove or disprove that $x\,y + y = y$ whenever x and y are Boolean variables.

3. How many different Boolean functions are there of degree 3 ?

4. Find the sum-of-products expansion of a Boolean function $f(x,y,z)$ that is 1 if and only if $x = y = 1$ and $z = 0$, or $x = 0$ and $y = z = 1$, or $x = y = 0$ and $z = 1$.

5. What is the output of the following circuit?



6. Use a K-map to minimize the sum-of-products expansion $xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z$.

7. Prove or disprove that $x + x\,y + x\,y\,z = x$ whenever x, y, and z are Boolean variables.

8. Find a Boolean function $f(x,y,z)$ that has the value 1 if and only if exactly two of x, y, and z have the value 1.

9. Is the set of operators $\{+,\ \cdot\}$ functionally complete? Justify your answer.

10. Construct a circuit using inverters, OR gates, and AND gates that gives an output of 1 if three people on a committee do not all vote the same.

11. Use a K-map to minimize the sum-of-products expansion $xyz + x\bar{y}z + \bar{x}\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$.